

Discrete Versions of Continuous Distributions and their Applications

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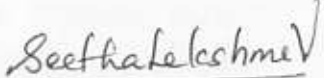
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CERTIFICATE

This is to certify that the thesis entitled "**Discrete Versions of Continuous Distributions and their Applications**" is a bonafide record of original research work carried out by **Ms. Simi Sebastian** under my supervision and guidance in the Department of Statistics, Nirmala College, Muvattupuzha, Ernakulam in partial fulfilment of the requirements for the Ph.D. degree of the Mahatma Gandhi University, Kottayam. I further certify that no part of this work has previously formed the basis for the award of any Degree, Diploma or other titles of any University or Society.

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Abstract

The present work mainly concentrates on study of discrete analogues of continuous distributions and their applications in time series modelling and reliability analysis. The discretized distributions derived from continuous distributions have similar properties to that of their continuous counterparts. These discretized versions have many applications mainly in communication, engineering and in finance.

Random variables which take values over $(-\infty, \infty)$ arise in many situations in time series modelling, reliability theory and in stress-strength analysis. But, probability distributions defined over \mathbb{Z} (including both positive and negative integers) are rare in literature. In the present work, we humbly attempt to introduce some probability distributions which are defined over \mathbb{Z} . These models are used to fit paired count data which usually arise in a medical field when before and after treatments are measured.

We introduce discrete Marshall-Olkin Fréchet distribution, generalized discrete Laplace distribution, discrete transmuted Weibull distribution and extended versions of discrete generalized exponential distribution. We use various inference procedures for estimating the parameters of the distributions under study. Integer valued auto regressive processes are developed for the models and applications of the distributions are illustrated through real data sets.

Chapter 1 is an introductory chapter which includes a brief introduction to the topic and review of literature. Chapter 2 deals with discrete Fréchet distribution and its generalizations. Discrete Marshall-Olkin Fréchet distribution is introduced and its important properties are studied. The parameters of the distribution are estimated using maximum likelihood method and the method of proportion of zero's and one's. Simulation study is carried out for evaluating the performance of the estimates and applications of the distribution are illustrated using real data sets.

A generalization of discrete Laplace distribution is introduced and its properties are studied in chapter 3. Representation of the proposed model as the difference of two negative binomial distribution is also established. The parameters of the distribution are estimated using maximum likelihood method and a simulation study is conducted. Application

of the model is illustrated using a real data. An extended distribution called geometric generalized discrete Laplace distribution is also introduced and its properties are studied.

In chapter 4, discrete generalized exponential distribution ($DGE(\alpha, p)$) and its applications are considered. A generalized version of $DGE(\alpha, p)$, called discrete generalized exponential distribution on \mathbb{Z} is introduced and its properties are studied. A distribution defined over \mathbb{Z} is also developed as a discrete analogue of generalized exponential distribution on real line, introduced by Jayakumar et al. (2012). These distributions have many applications in time series modelling and stress-strength analysis. Applications of the model are illustrated using a real data namely, DMFT index data (Bohning et al., 1999). An integer valued auto regressive process of order one is developed with $DGE(\alpha, p)$ distribution as marginals.

In chapter 5, integer valued auto regressive process is developed for observations over \mathbb{Z} . Here, the INAR(I) model developed for \mathbb{Z}^+ is extended to \mathbb{Z} using Pegram's operator and thinning operator. The sample path properties of the model are studied. The applications of the model is illustrated using a financial data from Saudi Stock market. R programs necessary for computation are also given in Appendix.

In chapter 6, a discrete analogue of transmuted Weibull distribution is introduced as a member of T-X family of distributions. The properties like mean, variance, hazard rate are studied and parameters of the distributions are estimated by maximum likelihood method and a simulation study is conducted. Also, a discrete version of Weibull distribution is introduced using reverse hazard rate function. The application of discretized distributions using RHR function in reliability is discussed.

Keywords: Autoregressive Processes, Binomial Thinning Operator, Extreme Value Distribution, Financial Modelling, Generalized Exponential Distribution, Generalized Laplace Distribution, Geometric Infinite Divisibility, Hazard Rate Function, INAR(1) Process, Marshall-Olkin Frechet Distribution, Order Statistics, Pegram's Mixing Operator, Reverse Hazard Rate, Survival Function, Time Series Modelling, Transmuted Weibull Distributions, T-X family of Distributions.

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1.1.7. Marginal Linn Family of Distributions	15
1.4. Summary of the Present Work	16
2. Discrete Marshall-Olkin Fréchet Distribution and Its Applications	21
2.1. Introduction	21
2.2. Discrete Marshall-Olkin Fréchet Distribution	22
2.2.1. Reliability Properties	22
2.2.2. Dependence Properties	22
2.2.3. Order Statistics	22
2.3. Estimation of Parameters of DMOF Distribution	23
2.4. Application to Real Data	24
2.5. Discrete Fréchet Distribution	25

Contents

<i>List of Tables</i>	v
<i>List of Figures</i>	vii

1 Introduction and Summary	1
1.1 Introduction	1
1.2 Review of Literature	3
1.2.1 Maximum Entropy Method	4
1.2.2 Discretization Using Survival Function	6
1.2.3 Discretization as T-geometric Family	9
1.2.4 Discretization Using Reverse Hazard Rate Function	11
1.2.5 Integer Valued Time Series Analysis	12
1.3 Basic Concepts	13
1.3.1 Discrete Self Decomposability	13
1.3.2 Infinite Divisibility	13
1.3.3 Geometric Infinite Divisibility	14
1.3.4 Thinning Operator	14
1.3.5 Pegram's Operator	15
1.3.6 INAR(1) Process	15

1.3.7	Marshall Olkin Family of Distributions	16
1.4	Summary of the Present Work	16
2	Discrete Marshall-Olkin Fréchet Distribution and its Applications	21
2.1	Introduction	21
2.1.1	Marshall-Olkin Fréchet Distribution	22
2.2	Discrete Marshall-Olkin Fréchet Distribution	24
2.2.1	Reliability Properties	25
2.2.2	Distributional Properties	27
2.2.3	Order Statistics	29
2.3	Estimation of Parameters of DMOF Distribution	30
2.4	Application to Real Data	31
2.5	Discrete Fréchet Distribution	33
2.5.1	Cumulative Distribution Function	33
2.5.2	Reliability Properties	33
2.5.3	Moments	36
2.5.4	Random Variable Generation	37
2.6	Estimation of Parameters of Discrete Fréchet Distribution	38
2.6.1	Method of Proportion of Zero's and One's	38
2.6.2	Method of Proportion of Zero's and Mean	39
2.6.3	Method of Moments	39
2.6.4	Maximum Likelihood Method	39
2.6.5	Simulation Study	40
2.6.6	Data Analysis	42
2.7	Conclusion	43
3	Generalized Discrete Laplace Distribution	45
3.1	Introduction	45
3.1.1	Discrete Laplace Distribution	47
3.2	Generalized Discrete Laplace Distribution	48
3.2.1	Probability Mass Function of $GDL(\beta, p_1, p_2)$	50

3.3	Properties of $GDL(\beta, p_1, p_2)$ Distribution	53
3.3.1	Moment Generating Function	53
3.3.2	Cumulative Distribution Function	55
3.3.3	Factorial Moments	56
3.3.4	Infinite Divisibility	56
3.3.5	Convolution Property	57
3.4	Estimation of the Parameters of $GDL(\beta, p_1, p_2)$ Distribution	57
3.4.1	Method of Moments	57
3.4.2	Method of Factorial Moments	58
3.4.3	Maximum Likelihood Estimation	58
3.4.4	Simulation Study	60
3.5	Application	61
3.6	Geometric Generalized Discrete Laplace Distribution ($GGDL$)	64
3.7	Conclusion	65
4	Discrete Generalized Exponential Distribution:Generalizations and Applications 67	
4.1	Introduction	67
4.1.1	Discrete Generalized Exponential Distribution (DGE)	68
4.2	Discrete Generalized Exponential Distribution on \mathbb{Z}	71
4.3	Estimation of the Parameters of $DGEZ(\alpha, p_1, p_2)$ Distribution	74
4.4	Data Analysis	75
4.5	Two Sided Discrete Generalized Exponential Distribution	76
4.6	Distributional Properties	77
4.6.1	Distribution Function	79
4.6.2	Moment Generating Function	80
4.7	An INARP(1) Process with $DGE(\alpha, p)$ Marginal Distribution	81
4.7.1	Properties of INARP(1) Process with $DGE(\alpha, p)$ Marginals	83
4.7.2	Parameter Estimation in INARP(1)	85
4.8	Conclusion	86

5	INAR(1) Processes with Generalized Discrete Laplace Marginals	87
5.1	Introduction	87
5.2	The Generalized Symmetric Discrete Laplace Marginal Distribution	89
5.3	INAR(1) Process with Negative Binomial Marginals	91
5.4	GDLINAR Process	92
5.4.1	Properties of $GDLINAR(1)$ Process	93
5.5	INARP(1) Processes	96
5.5.1	Properties of INARP(1) Process with $GDL(\beta, p)$ Marginals	98
5.5.2	Higher Order Process	101
5.5.3	Parameter Estimation in INARP(1)	102
5.5.4	Simulation Study	103
5.6	Application	103
5.7	Comparison Between $GDLINAR(1)$ and $INARP(1)$ Model	107
5.8	Conclusions	107
5.9	Appendix	109
6	Some Reliability Applications of Discretized Distributions	111
6.1	Introduction	111
6.1.1	Transmuted Weibull Distribution	112
6.2	Transmuted Weibull Geometric Distribution	114
6.2.1	Representation	115
6.3	Reliability Properties	117
6.3.1	Random Variable Generation	117
6.4	Estimation of Parameters in TWG distribution	119
6.5	Data Analysis	120
6.6	Discretization Using Reverse Hazard Rate Function	122
6.7	Applications	124
6.8	Conclusion	124
	References	126
	Index	137