Discrete Versions of Continuous Distributions and their Applications

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CERTIFICATE

This is to certify that the thesis entitled **"Discrete Versions of Continuous Distributions and their Applications"** is a bonafide record of original research work carried out by **Ms. Simi Sebastian** under my supervision and guidance in the Department of Statistics, Nirmala College, Muvattupuzha, Ernakulam in partial fulfilment of the requirements for the Ph.D. degree of the Mahatma Gandhi University, Kottayam. I further certify that no part of this work has previously formed the basis for the award of any Degree, Diploma or other titles of any University or Society.

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Abstract

The present work mainly concentrates on study of discrete analogues of continuous distributions and their applications in time series modelling and reliability analysis. The discretized distributions derived from continuous distributions have similar properties to that of their continuous counterparts. These discretized versions have many applications mainly in communication, engineering and in finance.

Random variables which take values over $(-\infty, \infty)$ arise in many situations in time series modelling, reliability theory and in stress-strength analysis. But, probability distributions defined over $\mathbb{Z}$ (including both positive and negative integers) are rare in literature. In the present work, we humbly attempt to introduce some probability distributions which are defined over $\mathbb{Z}$. These models are used to fit paired count data which usually arise in a medical field when before and after treatments are measured.

We introduce discrete Marshall-Olkin Fréchet distribution, generalized discrete Laplace distribution, discrete transmuted Weibull distribution and extended versions of discrete generalized exponential distribution. We use various inference procedures for estimating the parameters of the distributions under study. Integer valued auto regressive processes are developed for the models and applications of the distributions are illustrated through real data sets.

Chapter 1 is an introductory chapter which includes a brief introduction to the topic and review of literature. Chapter 2 deals with discrete Fréchet distribution and its generalizations. Discrete Marshall-Olkin Fréchet distribution is introduced and its important properties are studied. The parameters of the distribution are estimated using maximum likelihood method and the method of proportion of zero's and one's. Simulation study is carried out for evaluating the performance of the estimates and applications of the distribution are illustrated using real data sets.

A generalization of discrete Laplace distribution is introduced and its properties are studied in chapter 3. Representation of the proposed model as the difference of two negative binomial distribution is also established. The parameters of the distribution are estimated using maximum likelihood method and a simulation study is conducted. Application

of the model is illustrated using a real data. An extended distribution called geometric generalized discrete Laplace distribution is also introduced and its properties are studied.

In chapter 4, discrete generalized exponential distribution ($DGE(\alpha, p)$) and its applications are considered. A generalized version of $DGE(\alpha, p)$, called discrete generalized exponential distribution on $\mathbb{Z}$ is introduced and its properties are studied. A distribution defined over $\mathbb{Z}$ is also developed as a discrete analogue of generalized exponential distribution on real line, introduced by Jayakumar et al. (2012). These distributions have many applications in time series modelling and stress-strength analysis. Applications of the model are illustrated using a real data namely, DMFT index data (Bohning et al., 1999). An integer valued auto regressive process of order one is developed with $DGE(\alpha, p)$ distribution as marginals.

In chapter 5, integer valued auto regressive process is developed for observations over $\mathbb{Z}$. Here, the INAR(I) model developed for $\mathbb{Z}^+$ is extended to $\mathbb{Z}$ using Pegram's operator and thinning operator. The sample path properties of the model are studied. The applications of the model is illustrated using a financial data from Saudi Stock market. R programs necessary for computation are also given in Appendix.

In chapter 6, a discrete analogue of transmuted Weibull distribution is introduced as a member of T-X family of distributions. The properties like mean, variance, hazard rate are studied and parameters of the distributions are estimated by maximum likelihood method and a simulation study is conducted. Also, a discrete version of Weibull distribution is introduced using reverse hazard rate function. The application of discretized distributions using RHR function in reliability is discussed.

Keywords: Autoregressive Processes, Binomial Thinning Operator, Extreme Value Distribution, Financial Modelling, Generalized Exponential Distribution, Generalized Laplace Distribution, Geometric Infinite Divisibility, Hazard Rate Function, INAR(1) Process, Marshall-Olkin Frechet Distribution, Order Statistics, Pegram's Mixing Operator, Reverse Hazard Rate, Survival Function, Time Series Modelling, Transmuted Weibull Distributions, T-X family of Distributions.

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Simi Sebastian

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