

A Study on Non-Gaussian Time Series Modeling and Applications

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By

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CERTIFICATE

This is to certify that this thesis entitled **A Study on Non-Gaussian Time Series Modeling and Applications** is a bonafide record of original research work carried out by Smt. Seetha Lekshmi V. under my supervision and guidance in the P.G and Research Department of Statistics, St. Thomas College, Pala in partial fulfilment of the requirements for the Ph.D. Degree of the Mahatma Gandhi University, Kottayam. I further certify that no part of this work has previously formed the basis for the award of any Degree, Diploma or other titles of any University or Society.



Pala
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Dr. K.K. Jose
(Supervising Teacher)

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CONTENTS

Page

Chapter 1

Introduction and Summary

01

1.1	Non-Gaussian Time Series Modeling-A Review	01
1.2	Summary of the present work	06
1.3	Preliminary Concepts	10
1.3.1	Infinite divisibility	10
1.3.2	Self decomposability	11
1.3.3	Stable Distributions on \mathbb{R}	11
1.3.4	Geometric Infinite divisibility	12
1.3.5	Bernstein function	15
1.3.6	Geometric Stable distributions	16
1.3.7	Geometric strictly Stable distribution	19
1.3.8	Mittag-Leffler Distribution	19
1.3.9	α -Laplace distribution	20
1.3.10	Geometric Exponential Distribution	21
1.3.11	Generalized Laplacian distribution	22
1.3.12	Stationary Time Series	23
1.3.13	Time reversibility	23
1.3.14	Autoregressive models	24

Chapter 2

Geometric Exponential Distribution and Applications

25

2.1	Introduction	25
2.2	Geometric Exponential Distribution	25
2.3	Geometric Gamma Distribution	28
2.4	Application of GED (λ) and GGD (λ, k) distribution in first order autoregressive time series modeling	30
2.4.1	The distribution of sums of stationary intervals	32
2.4.2	The joint distribution of (X_n, X_{n-1})	33
2.5	Generalization to a k^{th} order autoregressive process	33
2.6	GED and Subordinated process	34
2.7	Graphs of density function and c.d.f.	35

Chapter 3		
Geometric Mittag-Leffler Distributions and Processes		40
3.1	Introduction	40
3.2	Geometric Mittag-Leffler distribution	41
3.3	Geometric Semi Mittag-Leffler distribution	44
3.4	Autoregressive models in geometric Mittag-Leffler marginals	45
3.5	Joint distribution of (X_n, X_{n-1})	47
3.6	Sums of stationary intervals	48
3.7	Characterization of geometric Mittag-Leffler distribution	49
3.8	Generalization to a k^{th} order autoregressive process	50
3.9	Applications	51
Chapter 4		
Geometric α-Laplace Distribution and Applications		52
4.1	Introduction	52
4.2	Semi α -Laplace distribution	53
4.3	Geometric Semi α -Laplace distribution	54
4.4	AR(1) models with Semi α -Laplace marginals	56
4.4.1	The Joint distribution of X_n and X_{n-1}	57
4.5	Geometric α -Laplace distribution	58
4.5.1	The joint distribution of X_n and X_{n-1}	60
4.6	Geometric Laplace distribution	60
4.6.1	The joint distribution of (X_n, X_{n-1})	64
4.7	Generalization to a k^{th} order autoregressive process	64
4.8	Applications	65
Chapter 5		
Autoregressive Processes with tailed Marginals		66
5.1	Introduction	66
5.2	Exponential tailed distribution	67
5.3	Geometric Exponential tailed distribution	68
5.4	Geometric gamma tailed distribution	68
5.5	Geometric Mittag-Leffler tailed distribution	68
5.6	Geometric Semi Mittag-Leffler tailed distribution	69

5.7	Geometric Semi α -Laplace tailed distribution	69
5.8	Autoregressive models with tailed marginal distribution	70
5.8.1	Geometric Exponential tailed autoregressive processes	71
5.8.2	Geometric gamma tailed autoregressive processes	72
5.8.3	Geometric Mittag-Leffler tailed autoregressive processes	74
5.8.4	Geometric Semi α -Laplace tailed autoregressive processes	75

References		79
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