CONCOMITANTS OF ORDER STATISTICS FROM BIVARIATE GENERALIZED MORGENSTERN AND CAMBANIS FAMILIES

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Declaration

*I do hereby declare that this thesis entitled "**Concomitants of Order Statistics from Bivariate Generalized Morgenstern and Cambanis Families**" is a bonafide record of the research work carried out by me during the course of research and that the thesis has not previously formed the basis for the award of any degree, diploma, associateship, fellowship or other similar title or recognition of any University or Society. I also declare that to the best of my knowledge and belief it contains no materials previously published by any person, except where due references are made in the text of the thesis.*

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Certificate

This is to certify that the thesis entitled "Concomitants of Order Statistics from Bivariate Generalized Morgenstern and Cambanis Families" is a record of bonafide research carried out by Biju Thomas under my supervision in the Department of Statistics, Nirmala College, Muvattupuzha and that no part of this work has formed the basis for the award of any Degree, Diploma or other similar titles of this or any other University or Society.

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Abstract and Keywords

Abstract: The theory and applications of order statistics are largely confined to the univariate data. The introduction of concomitants of order statistics by David in 1973 removed this restriction and paved the way for extending the results to the multivariate data. Let (X_i, Y_i), i = 1,2,…,n be a random sample from a bivariate distribution with cumulative distribution function (cdf) F(x,y). If the X-sample values are ordered, the Y-value paired with the r^{th} order statistic $X_{r:n}$ is its concomitant $Y_{[r:n]}$. In this thesis we address four problems related to the distribution theory and some aspects of concomitants of order statistics and record concomitants.

The first problem is about the distribution of concomitants of order statistics from the bivariate Generalized Morgenstern family specified by the cdf,

$$F_{X,Y}(x,y) = F_X(x)F_Y(y) + \alpha\left\{F_X(x)[1-F_X(x)]\right\}^m \left\{F_Y(y)[1-F_Y(y)]\right\}^p.$$

The constants 'm' and 'p' involved in this model are real numbers ≥ 1 and 'α' is a real constant constrained to lie in an interval about zero. In this work we use copulas to obtain expressions for Kendall's tau and Spearman's rho for this model. We obtain the mean, variance and higher moments of the r^{th} concomitant from this family. We specialize these results to an important member of this family, viz., bivariate uniform distribution. We also provide a quick estimator for the parameter of the expensive marginal variable of the bivariate uniform distribution and tabulate the variances of this estimator for different values of the parameters.

The second problem we explore is about the distribution theory of concomitants of order statistics from bivariate Cambanis system represented by the cdf,

$$F_{X,Y}(x,y) = F_X(x)F_Y(y)[1+\alpha_1\{1-F_X(x)\}+\alpha_2\{1-F_Y(y)\}+\alpha_3\{1-F_X(x)\}\{1-F_Y(y)\}],$$

where the parameters α_1, α_2 and α_3 satisfy the conditions

$$1+\alpha_1+\alpha_2+\alpha_3 \geq 0, \qquad 1+\alpha_1-\alpha_2-\alpha_3 \geq 0,$$

$$1-\alpha_1+\alpha_2-\alpha_3 \geq 0, \qquad 1-\alpha_1-\alpha_2+\alpha_3 \geq 0.$$

These conditions are satisfied by a convex set including the region $|\alpha_1|, |\alpha_2|, |\alpha_3| \leq \frac{1}{3}$.

The marginal distributions of X and Y will be respectively

$$G_X(x) = F_X(x)[1+\alpha_1\{1-F_X(x)\}],$$

$$G_Y(y) = F_Y(y)[1+\alpha_2\{1-F_Y(y)\}].$$

If $\alpha_1 = 0$, the marginal distribution of X reduces to $F_X(x)$ and hence it is easy to derive the conditional distribution of Y given X. As a consequence the derivation of the distribution of concomitants is much easier than the case $\alpha_1 \neq 0$. Accordingly we investigate the distribution theory of concomitants from bivariate Cambanis family when $\alpha_1 = 0$ and $\alpha_1 \neq 0$ separately. We specialize these results to the members of the family, viz., bivariate exponential and bivariate two-sided power (TSP) distribution (when $\alpha_1 = 0$) and Cambanis type Pareto distributions (when $\alpha_1 \neq 0$).Using copulas we obtain expressions for Kendall's tau, Spearman's rho and condition for total positive dependence of order two (TP_2) for this family. Variances and expected values of $Y_{[r:n]}$ for selected distributions are tabulated for different values of association parameters and for different values of 'r' and 'n'.

The n^{th} record value R_n corresponding to a sequence of independent and identically distributed random variables $X_1, X_2, ...$ with common cdf F(x) is that value X_j such that $X_j > X_i$ for all i = 1, 2,..., j-1. When we deal with a bivariate data, the Y value paired with the X value R_n is the n^{th} record concomitant $R_{[n]}$. The third problem we investigate is about the distribution theory of record concomitants from bivariate Cambanis family (when $\alpha_1 = 0$). We specialize these results to the members of the family, viz., bivariate uniform and bivariate gamma distributions. Employing copulas various dependence measures of concomitants of records from bivariate Cambanis family are derived. Expected values and variances of the record concomitant $R_{[n]}$ for the Cambanis type bivariate gamma distribution are tabulated.

Motivated from ranked set sampling, the fourth problem we examine is about extending the concept of concomitants of order statistics to higher order. Accordingly we introduce the concept of second order concomitants and their distribution theory is

studied in detail. An estimation procedure and a selection problem based on second order concomitants are also discussed.

Keywords: bivariate Cambanis family, bivariate exponential, bivariate gamma, bivariate Generalized Morgenstern family, bivariate Pareto, bivariate uniform, concomitants of order statistics, concomitants of records, copula, dependence measures, distribution theory, estimation, Kendall's tau, Morgenstern family, order statistics, quasi range, ranked set sampling, records, second order concomitants, selection problem, Spearman's rho, total positive dependence of order two (TP_2),two-sided power (TSP) distribution.

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