SOME ASPECTS OF CONCOMITANTS OF ORDER STATISTICS

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Dr. JOHNY SCARIA
(Research Guide)

Abstract and Keywords

In modelling bivariate data, when the prior information is in the form of marginal distributions, it is an advantage to consider families of bivariate distributions with specified marginals. Upon realising that the Morgenstern system provides a flexible family that can be used in such contexts, Scaria and Nair (1999) have introduced the distribution theory of concomitants of order statistics from the Morgenstern family. This research work concentrates on the study of concomitants of order statistics from the Cambanis and Morgenstern bivariate families. In this thesis we deal with six problems related to distribution theory, applications and some aspects of concomitants and record concomitants of order statistics.

The first problem is about the distributional characteristics and reliability properties of the Cambanis family specified by the cumulative distribution function (cdf),

$$F(x_1,x_2) = F_1(x_1)F_2(x_2)[1+\lambda_1(1-F_1(x_1))+\lambda_2(1-F_2(x_2))+\lambda_3(1-F_1(x_1))(1-F_2(x_2))],$$

where the parameters satisfy the conditions

$$(1+\lambda_1+\lambda_2+\lambda_3) > 0, \quad (1-\lambda_1-\lambda_2+\lambda_3) > 0,$$
$$(1-\lambda_1+\lambda_2-\lambda_3) > 0 \text{ and } (1+\lambda_1-\lambda_2-\lambda_3) > 0,$$

such that λ's are real constants. We also demonstrate the application of the family in modelling bivariate lifetime data.

Let (X_i, Y_i), $i = 1, 2, \ldots, n$ be a random sample from a bivariate distribution with cdf $F(x, y)$. If the X-sample values are ordered, the Y value paired with the r^{th} order statistic $X_{r:n}$ is its concomitant $Y_{[r:n]}$. The second problem that is being explored is about the dependence measures and information measures including Shannon entropy and Kullback-Leibler distance of the concomitants of order statistics from the Cambanis family. The distribution theory of the concomitants from the Cambanis type bivariate logistic and exponentiated exponential distributions are considered. Additionally, some recurrence relations connecting the moments have also been derived. Further, the Kullback-Leibler distance between the concomitants of the $(\frac{n+1}{2})^{th}$ and r^{th} order statistics and the Shannon entropy values have been tabulated.

Let (X_i, Y_i), i= 1, 2,... be a sequence of independent and identically distributed random variable from an absolutely continuous distribution with cdf $F(x, y)$. Let R_n denote the n^{th} record value in the sequence of the X's. Then the Y-value paired with the X-value R_n is called the n^{th} record concomitant and will be denoted by $R_{[n]}$. The third problem we investigate is about the information measures of record concomitants from the Cambanis type bivariate distributions. The Kullback-Leibler distance between the record concomitants is also evaluated. The distribution theory and Shannon entropy measure of record concomitants from Cambanis type bivariate logistic and exponentiated exponential distribution are discussed.

Inspired by the concept of second order concomitants and their distribution theory introduced by Scaria and Thomas (2014), the fourth problem we examine is about the distribution theory of second order concomitants from the Morgenstern type bivariate logistic, exponential and gamma distribution. Some recurrence relations between the moments of second order concomitants from the logistic and exponential distribution are derived. The dependence structure of the second order concomitants from the Morgenstern family are also studied.

Let $F_{Y_{[r:n]}}$ and $F_{Y_{[s:n]}}$ denote the distribution function of the component lifetimes of the concomitants of order statistics $X_{r:n}$ and $X_{s:n}$ respectively. By considering T_1 = min $(Y_{[r:n]}, Y_{[s:n]})$ and T_2 = max $(Y_{[r:n]}, Y_{[s:n]})$ respectively as the lifetimes of the series and parallel systems of two components, the fifth problem we explore is about the distribution theory of lifetimes of two component systems. The dependence concepts of concomitants from the Morgenstern family are studied. An application of concomitants in reliability modelling of designing a two component system from the Morgenstern type bivariate distributions such as exponentiated exponential and gamma distribution are presented. The mean times to failure of two component systems (series and parallel) using concomitants are tabulated.

The sixth problem we investigate is about the characterizations of the bivariate distribution function through properties relevant to stress-strength models, and the regression functions. The distribution theory of the concomitants of the extended Farlie-Gumbel-Morgenstern family of bivariate distributions are also studied. The study attempt characterization of the bivariate distribution function through properties useful in the study of material strength, average lifetimes of the longest living components and the regression functions using the concomitants of order statistics.

Keywords: Association measures, Bivariate Cambanis family, Bivariate exponential, Bivariate exponentiated exponential, Bivariate gamma, Bivariate hazard rates, Bivariate logistic, Bivariate mean residual life, Bivariate Morgenstern family, Characterizations, Concomitants of order statistics, Extended Farlie-Gumbel-Morgenstern distribution, Kullback-Leibler distance, Mean times to failure, Record concomitants, Reliability, Second order concomitants, Series and parallel systems, Shannon entropy, Total positivity.

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Sithara Mohan

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